NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2499

LAMINAR FRICTION AND HEAT TRANSFER AT MACH NUMBERS FROM 1 TO 10

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SUMMARY

A method of solving the laminar boundary-layer equations for compressible flow, in the absence of a pressure gradient, is developed without imposing restrictions on the thermal properties of the fluid medium. Velocity and temperature profiles and boundary-layer characteristics have been computed for Mach numbers from 1 to 10, utilizing experimental values of the heat capacity, viscosity, and conductivity. The analysis shows that an effective temperature, which is a function of the surface temperature and stream conditions and is similar to the recovery temperature, arises naturally and is the proper reference temperature to be used in heat-transfer calculations. The effective temperature and the recovery temperature become identical for the condition of zero heat transfer. The recovery factor and the analogous effective-temperature function decrease substantially with increasing values of Mach number. Thus, the high-speed aerodynamic heating problem is not as severe as is indicated by simpler theories.

INTRODUCTION

The large extent of laminar flow expected on missiles and aircraft operating at high speeds and altitudes makes an accurate knowledge of the laminar-flow frictional effects and heat-transfer characteristics desirable for practical design considerations. The calculation of the boundary-layer characteristics at high speeds is complicated by the necessity of considering simultaneously the compressibility of the fluid medium and the variation of thermal properties with temperature. As a consequence of the compressible nature of the flow, the temperature variation throughout the boundary layer is large and the variation of the thermal properties of the fluid is significant. At a stream Mach number of 5, for example, the ratio of maximum boundary-layer temperature (absolute) to stream temperature is of the order of 5 to 1 for the condition of zero heat transfer at the surface. Moreover, for these conditions, experiment shows that the viscosity and conductivity increase by factors of approximately 3 and 4 to 1, respectively, over their values at the edge of the boundary layer, while the heat capacity increases by approximately 16 percent and the Prandtl number decreases more than 10 percent. Thus, since the temperature variation within the boundary layer at high supersonic speeds is large, assumptions or restrictions regarding the thermal properties can conceivably have a significant influence on the calculated characteristics of the boundary layer.

The calculation of the characteristics of the laminar boundary layer at high speeds has been considered by several investigators (references 1 to 4, for example). In these and similar analyses, however, certain restrictions are imposed on the thermal properties of the fluid. Two assumptions are generally made: First, the heat capacity and the Prandtl number are considered to be independent of temperature; and, second, some simple law for the variation of viscosity (or conductivity) with temperature is specified. The constancy of the heat capacity and Prandtl number implies that the viscosity and conductivity vary with temperature in the same manner. For air, these conditions are nearly satisfied for small temperature variations. The viscosity-temperature relation is frequently taken as the equation $\frac{\mu}{\mu_1} = \left(\frac{T}{T_1}\right)^{\omega}$ where ω is a constant, a linear relation between viscosity and temperature, or the Sutherland equation. Of these three, the Sutherland equation especially gives good results for the viscosity over a wide temperature range, although it is not as satisfactory for the conductivity. The choice of a linear viscosity-temperature relation (reference 4, for example) or a Prandtl number of unity (reference 1, for example) leads to an essential simplification, since in each case only one differential equation need be solved. Although these various restrictions on the thermal characteristics appear justified for small temperature variations, they are not justified a priori for the calculation of boundary-layer characteristics at high Mach numbers where the temperature variations are large.

The present investigation is concerned with the calculation of the characteristics of the laminar boundary layer in the absence of a pressure gradient at supersonic speeds. The purpose of the investigation is twofold: first, to determine the characteristics of laminar boundary layers at supersonic Mach numbers and, second, to provide information which may be used to evaluate restrictions frequently imposed on the thermal characteristics. The method developed herein places no restrictions on the thermal properties of the fluid. Experimental values for the variation of heat capacity, conductivity, viscosity, and Prandtl number have been employed in the calculations in order to obtain the flow properties as accurately as possible. The method can be adapted to the calculation of boundary layers with suction or blowing.

Computations have been made for a surface temperature equal to the stream temperature at Mach numbers from 1 to 10 and for an insulated surface for Mach numbers from 1 to 5. In addition, calculations have been made at a Mach number of 7 for ratios of surface temperature to stream temperature of 2 and 4. The stream temperature was taken as -67° F, a value that corresponds to the isothermal region of the NACA standard atmosphere.

The method of solution is essentially one of successive approximation. Ten iterations were required in most cases to obtain a difference of less than 1 percent, for both the shear stress and the heat-transfer rate at the surface, between successive approximations. Even with the use of automatic computing equipment (the Bell Telephone Laboratories X-66744 relay computer at the Langley Laboratory was used) appreciable time is required for the solution. As a consequence, the number of computations completed thus far is inadequate for a critical comparison of calculated boundary-layer characteristics with other analyses.

SYMBOLS

x,y coordinates parallel and normal to stream direction

ψ stream function

$$\zeta = \frac{\psi}{\sqrt{\rho_1 \mu_1 U_1 x}}$$

$$\eta = y\sqrt{\frac{U_1}{\nu_1 x}}$$

u,v components of velocity along x- and y-axes

U_1 stream velocity

M Mach number

T temperature

ϑ temperature function

ρ density

c_p	heat capacity at constant pressure
γ	ratio of heat capacities at constant pressure and constant volume
μ	coefficient of viscosity
ν	kinematic viscosity
k	thermal conductivity
q	local heat-transfer rate
h	local heat-transfer coefficient
R	Reynolds number
Pr	Prandtl number
Nu	Nusselt number
τ	shear stress
D	skin friction
C_f	skin-friction coefficient
α,β	integrating factors

$f = \rho^*\mu^*u^*$

$$\lambda = \frac{c_p^*}{Pr}\rho^*\mu^*u^*$$

F_1	function defined by equation (13)
F_2	function defined by equation (16)
g	function defined by equation (16)
φ	function defined by equation (14)
Φ	function defined by equation (17)

Subscripts:

1	free stream or at edge of boundary layer
e	effective
r	recovery
s	surface

Superscript:

*	dimensionless quantity based on stream conditions

ANALYSIS

Basic equations.- With the x-axis taken in the free-stream direction, the two-dimensional compressible-flow boundary-layer equations for steady motion in the absence of a pressure gradient may be written as follows:

Momentum equation:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{1}$$

Continuity equation:

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0 \tag{2}$$

Energy equation:

$$c_p \rho u \frac{\partial T}{\partial x} + c_p \rho v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are the velocity components in the direction of the x- and y-axes, respectively, ρ is the density, T is the absolute temperature, c_p is the heat capacity at constant pressure, μ is the coefficient of viscosity, k is the thermal conductivity, and the thermal properties are functions of the temperature.

The set of partial-differential equations (1) to (3) may be reduced to two ordinary differential equations for the particular cases of constant temperature or zero heat transfer at the surface $y = 0$. In order to accomplish this reduction it is convenient first to introduce the stream function as a new independent variable. The velocity components are related to the stream function by the equations:

$$\left.\begin{aligned} \rho u &= \frac{\partial\psi}{\partial y} \\ \rho v &= -\frac{\partial\psi}{\partial x} \end{aligned}\right\} \tag{4}$$

Then by taking x and ψ as the independent variables, equations (1) and (3) may be transformed into

$$\frac{\partial}{\partial\psi}\left(\rho\mu u\,\frac{\partial u}{\partial\psi}\right) - \frac{\partial u}{\partial x} = 0 \tag{5}$$

$$\frac{\partial}{\partial\psi}\left(k\rho u\,\frac{\partial T}{\partial\psi}\right) - c_p\,\frac{\partial T}{\partial x} = -\rho\mu u\left(\frac{\partial u}{\partial\psi}\right)^2 \tag{6}$$

and equation (2) is satisfied identically by virtue of equation (4).

It is found that surfaces of constant $\psi/\sqrt{x}$ are both isotherms and surfaces of constant velocity. Thus, with the choice of an independent variable proportional to $\psi/\sqrt{x}$, the equations of motion and energy may be reduced to ordinary differential equations for boundary conditions of constant temperature or zero heat transfer at the surface $y = 0$. By introduction of the independent variable $\zeta = \frac{\psi}{\sqrt{\rho_1\mu_1U_1x}}$ and the nondimensional temperature function ϑ, equations (5) and (6) become

$$\frac{d}{d\zeta}\left(\rho^*\mu^*u^* \frac{du^*}{d\zeta}\right) + \frac{\zeta}{2}\frac{du^*}{d\zeta} = 0 \tag{7}$$

$$\frac{d}{d\zeta}\left(\frac{c_p^*}{Pr} \rho^*\mu^*u^* \frac{d\vartheta}{d\zeta}\right) + c_p^* \frac{\zeta}{2}\frac{d\vartheta}{d\zeta} = -2\rho^*\mu^*u^*\left(\frac{du^*}{d\zeta}\right)^2 \tag{8}$$

where $u^* = \frac{u}{U_1}$, $\rho^* = \frac{\rho}{\rho_1}$, $T^* = \frac{T}{T_1}$, $\mu^* = \frac{\mu}{\mu_1}$, $c_p^* = \frac{c_p}{c_{p_1}}$, $Pr = \frac{c_p\mu}{k}$, and

$$\vartheta = \frac{T^* - 1}{\frac{U_1^2}{2c_{p_1}T_1}} = \frac{T^* - 1}{\frac{\gamma - 1}{2} M_1^2}$$

The symbol γ represents the ratio of heat capacities at constant pressure and constant volume, U_1 is the stream velocity, and the subscript 1 refers to values at the edge of the boundary layer or values in the free stream.

Von Mises (reference 5) employed ψ as an independent variable in the boundary-layer equations for incompressible flow and many authors since have utilized this transformation for various compressible-flow problems. The transformations employed herein follow those of Von Kármán and Tsien (reference 1).

The solution of the boundary-layer equations for compressible flow may be simplified in two cases where the heat capacity and Prandtl number are taken as constants. For a Prandtl number of unity, the temperature and velocity have a simple parabolic relation and only the differential equation of motion need be solved to determine the flow throughout the boundary layer (reference 6). For a linear viscosity-temperature relation the equation of motion is independent of the energy equations and may be reduced to the form occurring in incompressible flow. (See reference 4, for example.) Thus only the energy equation need be solved, since the incompressible-flow solutions are known. Where no restrictions are placed on the heat capacity, viscosity, conductivity, and Prandtl number, both equations (7) and (8) must be solved to determine the flow properties in the boundary layer.

Although the nonlinear equations (7) and (8) do not yield to a direct analytical solution, they can be solved numerically by a method of successive approximations. Since c_p^*, μ^*, ρ^*, and Pr are functions only of the temperature, and thus functions of ζ, the coefficients of the derivatives in equations (7) and (8) may be considered as known functions if some initial approximate solution is given. Then, with the quantities

$$f(\zeta) = \rho^*\mu^*u^*$$

and

$$\lambda(\zeta) = \frac{c_p^*}{\mathrm{Pr}} \rho^*\mu^*u^*$$

considered as known functions of ζ, and with the integrating factors

$$\alpha = \exp\left(\frac{1}{2}\int_0^\zeta \frac{\zeta}{f}\, d\zeta\right)$$

and

$$\beta = \exp\left(\frac{1}{2}\int_0^\zeta \mathrm{Pr}\, \frac{\zeta}{f}\, d\zeta\right)$$

the equation of motion (7) and the energy equation (8) may be written as

$$\frac{d}{d\zeta}\left(\alpha f\, \frac{du^*}{d\zeta}\right) = 0 \tag{9}$$

$$\frac{d}{d\zeta}\left(\beta\lambda\, \frac{d\vartheta}{d\zeta}\right) = -2\beta f\left(\frac{du^*}{d\zeta}\right)^2 \tag{10}$$

Solution of boundary value problem.- Two problems are considered herein. The first is to determine the flow throughout the laminar boundary layer subject to the condition of a constant surface temperature, and the second is to determine the flow for the condition of zero heat transfer at the surface. The boundary conditions to be satisfied for these two problems are:

For constant surface temperature:

At the surface ($\zeta = 0$), $u^* = 0$ and $\vartheta = \vartheta_s =$ Constant

At infinity ($\zeta \longrightarrow \infty$), $u^* = 1$ and $\vartheta = 0$

For zero heat transfer at the surface:

At the surface ($\zeta = 0$), $u^* = 0$ and

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = \text{Constant}\left(\rho^* u^* \frac{\partial \vartheta}{\partial \zeta}\right)_{\zeta=0} = 0$$

At infinity ($\zeta \longrightarrow \infty$), $u^* = 1$ and $\vartheta = 0$

The subscript s denotes the values at the surface.

Since the boundary conditions on u^* are the same for both problems, the solutions are of the same form.

The first integral of equation (9) is

$$\alpha f \frac{du^*}{d\zeta} = F_1 \tag{11}$$

and a second integration gives the velocity as

$$u^* = F_1 \int_0^{\zeta} \frac{d\zeta}{\alpha f} \tag{12}$$

From the boundary condition at infinity the function F_1 is found to be

$$F_1\left(M_1, T_1, T_s\right) = \left(\int_0^{\infty} \frac{d\zeta}{\alpha f}\right)^{-1} \tag{13}$$

The velocity distribution is thus determined from equations (11) to (13) for a given surface temperature (or for zero heat transfer at the surface) and for given stream conditions.

The temperature distribution corresponding to a constant value of the surface temperature is obtained by integrating equation (10). The first integral is

$$\beta\lambda \frac{d\vartheta}{d\zeta} = F_2 - \varphi(\zeta) \tag{14}$$

where

$$\varphi(\zeta) = 2\int_0^\zeta \beta f\left(\frac{du^*}{d\zeta}\right)^2 d\zeta$$

After a second integration the temperature function is found to be

$$\vartheta - \vartheta_s = F_2\int_0^\zeta \frac{d\zeta}{\beta\lambda} - \int_0^\zeta \frac{\varphi}{\beta\lambda}\, d\zeta \tag{15}$$

From the boundary condition at infinity, the function F_2 is determined as

$$F_2 = g\left(\vartheta_e - \vartheta_s\right) \tag{16}$$

where

$$g\left(M_1,T_1,T_s\right) = \left(\int_0^\infty \frac{d\zeta}{\beta\lambda}\right)^{-1}$$

$$\vartheta_e\left(M_1,T_1,T_s\right) = \int_0^\infty \frac{\varphi}{\beta\lambda}\, d\zeta$$

The temperature distribution corresponding to zero heat transfer at the surface $\zeta = 0$ is found in a similar manner. The first integral of equation (10) satisfying the conditions of zero heat transfer is

$$\beta\lambda \frac{d\vartheta}{d\zeta} = -\Phi(\zeta) \tag{17}$$

where

$$\Phi(\zeta) = 2\int_0^{\zeta} \beta f\left(\frac{du^*}{d\zeta}\right)^2 d\zeta$$

and after a second integration the temperature function is found to be

$$\vartheta = \int_{\zeta}^{\infty} \frac{\Phi}{\beta\lambda}\, d\zeta \tag{18}$$

For $\zeta = 0$, ϑ becomes the recovery factor

$$\vartheta_r = \int_0^{\infty} \frac{\Phi}{\beta\lambda}\, d\zeta$$

Equations (12) and (15) give the velocity and temperature distributions as functions of ζ corresponding to a specified surface temperature, whereas equations (12) and (18) give the velocity and temperature distributions as functions of ζ corresponding to zero heat transfer at the solid surface. These solutions may be expressed in terms of the more familiar independent variable $\eta = y\sqrt{\frac{U_1}{\nu_1 x}}$ (where ν is the kinematic viscosity) by means of the relation

$$\frac{d\zeta}{d\eta} = \rho^* u^*$$

or

$$\eta = y\sqrt{\frac{U_1}{\nu_1 x}} = \int_0^{\zeta} \frac{d\zeta}{\rho^* u^*}$$

found from the expressions defining η and ζ and from equation (4).

Comments on method of solution.- The solution of the equation of motion and the energy equation described herein imposes no restrictions on the variation of the material properties of the fluid - heat capacity, viscosity, and conductivity - with temperature. Since the method of solution is essentially one of numerical integration, experimentally determined values of these parameters or values found from analytical expressions may be used with equal facility. The equations developed herein may be simplified somewhat, with an attendant reduction in computational labor, for the particular case in which the heat capacity and the Prandtl number Pr are taken as constants. With these restrictions the function λ reduces to f/Pr and the integrating factor β reduces to α^{Pr}.

The solution of the equations for the velocity and temperature distributions throughout the boundary layer is a step-by-step method of successive approximations. Thus, initial approximate solutions for the velocity and temperature distributions are required. These initial solutions may conveniently be taken as those for $Pr = 1$ and $\rho^*\mu^* = 1$ or, better still, the solutions given in reference 4. These values are used to compute the quantities f, λ, α, and β as functions of ζ, and the velocity and temperature distributions are found by numerical integration. With these new values for the velocity and temperature distribution, the functions f, λ, α, and β are recalculated and the second approximation to the velocity and temperature distributions is determined. The process is continued until the desired accuracy is obtained. This method of solution is essentially the same as the one employed for evaluating the velocity distribution in reference 1.

RESULTS AND DISCUSSION

The method described in the preceding section has been employed to calculate a number of velocity and temperature profiles and boundary-layer characteristics at supersonic Mach numbers. Faired experimental values of c_p, μ, and k primarily from reference 7 were used in the computations in order to obtain reliable results at the high Mach numbers considered. The free-stream temperature T_1 was taken as 392.7 Rankine (-67° F), the value at the isothermal level of the standard atmosphere, and the corresponding values of the thermal properties were taken as:

$$c_{p_1} = 7.718 \text{ Btu/(slug)(deg)}$$

$$\mu_1 = 3.058 \times 10^{-7} \text{ slug/(ft)(sec)}$$

$$k_1 = 3.227 \times 10^{-6} \text{ Btu/(ft)(sec)(deg)}$$

$$Pr_1 = 0.73$$

The calculated velocity and temperature profiles for $T_s^* = 1$ at Mach numbers from 1 to 10 are presented in figure 1, and some results at a constant Mach number of 7 for values of $T_s^* = 1$, 2, and 4 are given in figure 2. Profiles for the condition of zero heat transfer at the wall are shown in figure 3. Although the calculated temperatures are very high at high Mach numbers, it is significant that these values are considerably lower than those predicted by simpler theories. Also of interest is the fact that the velocity profiles for the condition of zero heat transfer are nearly linear, whereas the profiles for a constant surface temperature have appreciable curvature. The velocity and temperature approach their stream values at smaller values of η for high Mach numbers than those predicted by simpler calculations; thus, a thinner boundary layer is indicated.

The frictional and heat-transfer characteristics may be evaluated with the aid of the equations developed in the analysis. The shear stress at the wall is given by

$$\tau_s = \left(\mu \frac{\partial u}{\partial y}\right)_{y=0} = \frac{\rho_1 U_1^2}{\sqrt{R}}\left(\rho^* \mu^* u^* \frac{du^*}{d\zeta}\right)_{\zeta=0}$$

where R is the Reynolds number $U_1 x/\nu_1$. From the equation defining α and from equation (11),

$$\left(\rho^* \mu^* u^* \frac{du^*}{d\zeta}\right)_{\zeta=0} = F_1$$

and the local skin friction may be written as

$$\tau_s = \frac{\rho_1 U_1^2}{\sqrt{R}} F_1 \qquad (19)$$

The friction drag may be found by integration as

$$D = \int_0^x \tau_s \, dx = 2 \frac{\rho_1 U_1^2}{\sqrt{R}} x F_1$$

and the average skin-friction coefficient then is given by

$$C_f = \frac{D}{\frac{1}{2}\rho_1 U_1{}^2 x} = \frac{4F_1}{\sqrt{R}} \tag{20}$$

The skin-friction coefficient found from equation (20) is shown in figure 4 for the conditions of constant surface temperature and zero heat transfer. This figure shows that the skin friction is significantly influenced by both the Mach number and the thermal state at the surface. The skin friction decreases with increasing Mach number, and at a given Mach number the skin friction increases with decreasing values of $T_s{}^*$. At a Mach number of 5 for example, the skin friction decreases from the Mach number 1 value by approximately 18 percent for zero heat transfer, and for a value of $T_s{}^* = 1$ the decrease is approximately 7 percent.

The convective heat-transfer rate at the surface per unit area (q) is given by the expression

$$q = -\left(k\frac{\partial T}{\partial y}\right)_{y=0} = -k_s\frac{U_1{}^2}{2c_{p_1}}\sqrt{\frac{U_1}{\nu_1 x}}\left(\rho^* u^*\frac{d\vartheta}{d\zeta}\right)_{\zeta=0}$$

and from equations (14) and (16), since $\beta = 1$ and $\varphi = 0$ for $\zeta = 0$,

$$\left(\rho^* u^*\frac{d\vartheta}{d\zeta}\right)_{\zeta=0} = \frac{c_{p_1}\mu_1}{k_s}\frac{g}{\frac{U_1{}^2}{2c_{p_1}}}\left(T_e - T_s\right)$$

The local heat-transfer rate then may be written as

$$q = -gc_{p_1}\sqrt{\frac{\rho_1\mu_1 U_1}{x}}\left(T_e - T_s\right) = -gc_{p_1}T_1\sqrt{\frac{\rho_1\mu_1 U_1}{x}}\left(T_e{}^* - T_s{}^*\right) \tag{21}$$

The heat-transfer rate is often expressed as the product of a heat-transfer coefficient h and a temperature increment. The temperature increment $T_e - T_s$ arises naturally in equation (21) and with this temperature difference an effective heat-transfer coefficient may be defined as

$$h_e = -gc_{p_1}\sqrt{\frac{\rho_1\mu_1U_1}{x}} \tag{22}$$

The usual temperature increment used for defining the heat-transfer coefficient is the difference between the recovery temperature and the surface temperature. With this temperature increment the heat-transfer coefficient becomes

$$h = -gc_{p_1}\sqrt{\frac{\rho_1\mu_1U_1}{x}}\,\frac{T_e{}^* - T_s{}^*}{T_r{}^* - T_s{}^*} = h_e\,\frac{T_e{}^* - T_s{}^*}{T_r{}^* - T_s{}^*} \tag{23}$$

The relative magnitudes of the temperatures T_r and T_e may be determined from the recovery factor ϑ_r and the effective-temperature function ϑ_e given in figure 5. Two points are significant: First, the recovery factor is considerably smaller than the effective-temperature function; second, both ϑ_r and ϑ_e decrease with increasing Mach number. The value of the recovery factor is usually considered to be approximately 0.85 for laminar flow. Figure 5 shows that for a Mach number of 5 the recovery factor is 0.766 and the effective-temperature function for $T_s{}^* = 1$ is 0.822. These values represent a sizable reduction in the recovery temperature T_r and the effective temperature T_e. Thus, the aerodynamic heating characteristics at high Mach numbers do not appear to be as severe as is indicated by simpler theories.

The equations defining ϑ_r and ϑ_e are of the same form. The only difference between the two expressions is that the parameters occurring in ϑ_e correspond to the condition of $T_s{}^* = \text{Constant}$, whereas the values in the expression for ϑ_r correspond to the condition of zero heat transfer. The functions ϑ_r and ϑ_e become identical for the condition of zero heat transfer. Since the temperature difference $T_e - T_s$ arises naturally in equation (21) for the heat-transfer rate, it appears that the proper reference temperature to be used for heat-transfer characteristics is T_e rather than T_r.

With the heat-transfer coefficients given by equations (22) and (23), two Nusselt numbers may be defined as

$$\left.\begin{aligned} Nu_e &= \frac{h_e x}{k_1} = -gPr_1\sqrt{R} \\ Nu &= \frac{hx}{k_1} = -gPr_1\sqrt{R}\,\frac{T_e^* - T_s^*}{T_r^* - T_s^*} = Nu_e\,\frac{T_e^* - T_s^*}{T_r^* - T_s^*} \end{aligned}\right\} \qquad (24)$$

Figure 6 shows that the quantity $|Nu|$ increases with increasing Mach number, whereas $|Nu_e|$ decreases. The Nusselt number $|Nu_e|$ decreases with increasing Mach number in direct proportion to the function g. The quantity $|Nu|$ increases with Mach number, however, since the quantity $\dfrac{T_e^* - T_s^*}{T_r^* - T_s^*}$ increases faster than g decreases. There is little change in Nu_e at Mach numbers above 7, since the heat-transfer coefficient h_e changes very little. The change of Nu or Nu_e over the entire Mach number range is not large.

From equations (19) and (21) a simple relation between the heat-transfer rate and the shear stress is found:

$$\frac{q}{\tau_s} = -\frac{g}{F_1}\,\frac{c_{p_1}T_1}{U_1}\left(T_e^* - T_s^*\right)$$

This relation, together with the heat-transfer rate from equation (21), is shown in figure 7. Since the skin friction decreases and $|q|$ increases with Mach number, the ratio $|q/\tau_s|$ increases with Mach number for a given value of T_s^*. The calculations at a Mach number of 7 indicate that the effect of increasing values of T_s^* is to decrease both the heat-transfer rate $|q|$ and the shear stress τ_s, and the ratio $|q/\tau_s|$ also decreases.

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National Advisory Committee for Aeronautics
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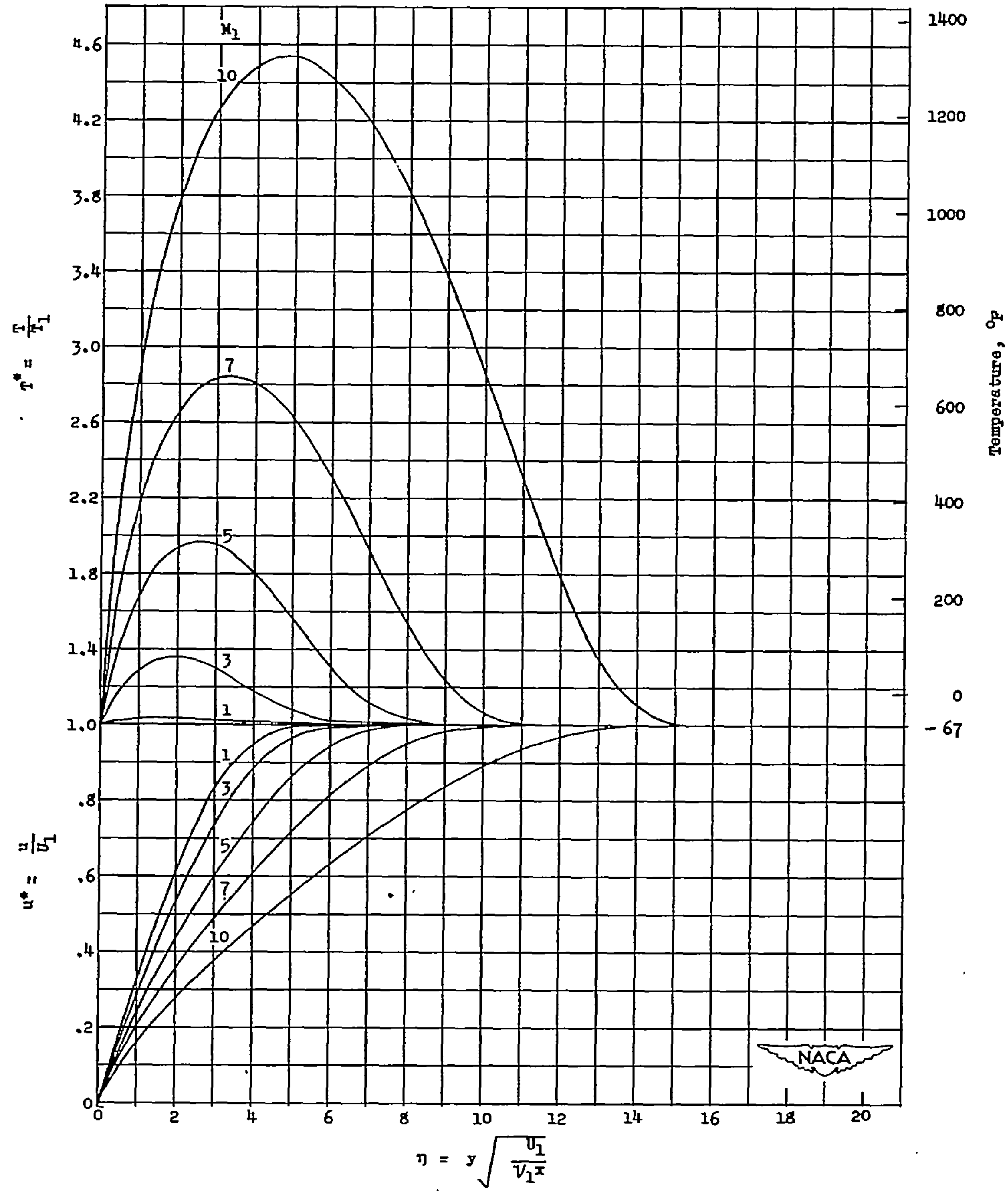

Figure 1.- Velocity and temperature profiles for $T_s^* = 1$.

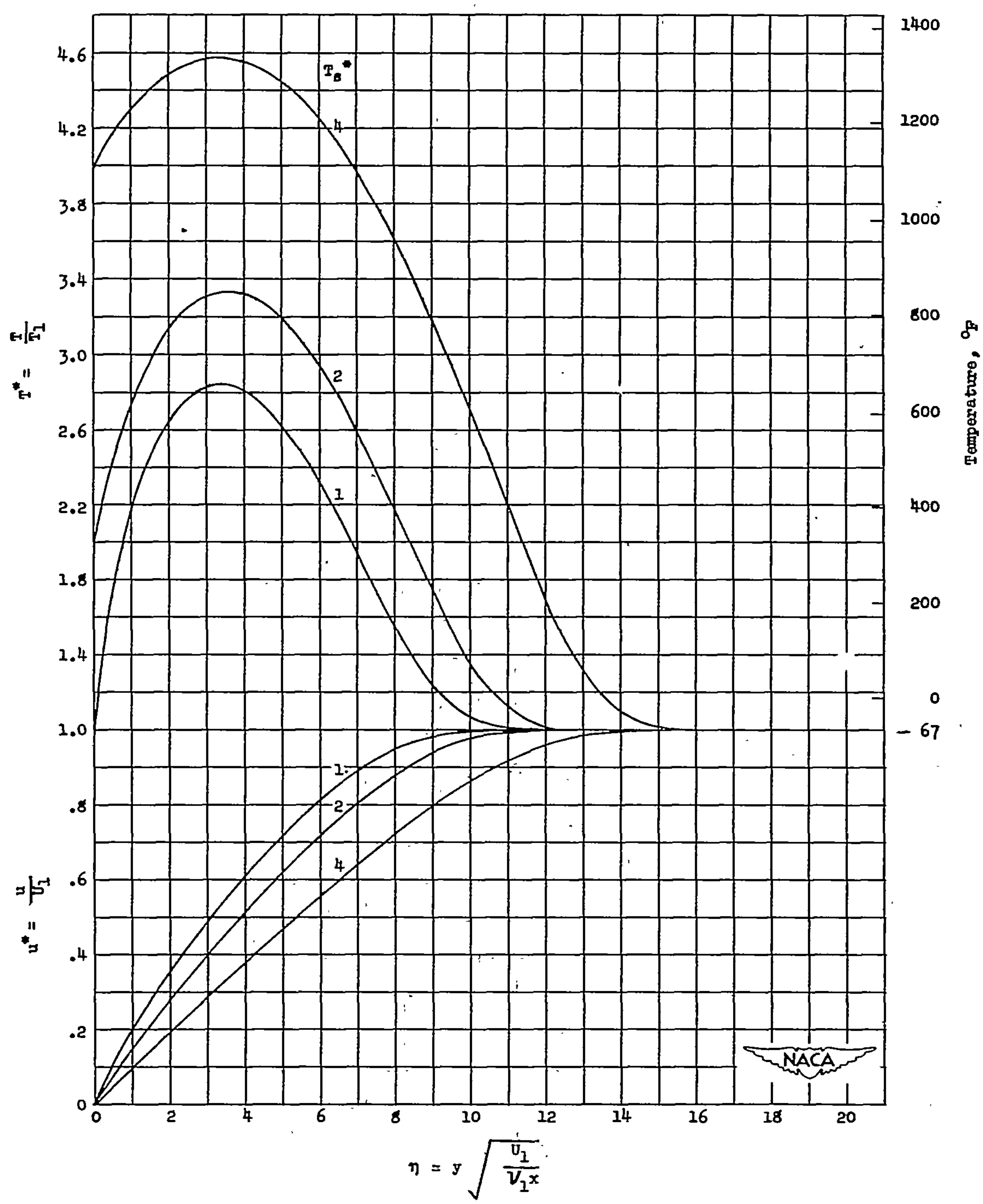

Figure 2.- Velocity and temperature profiles for $M_1 = 7$.

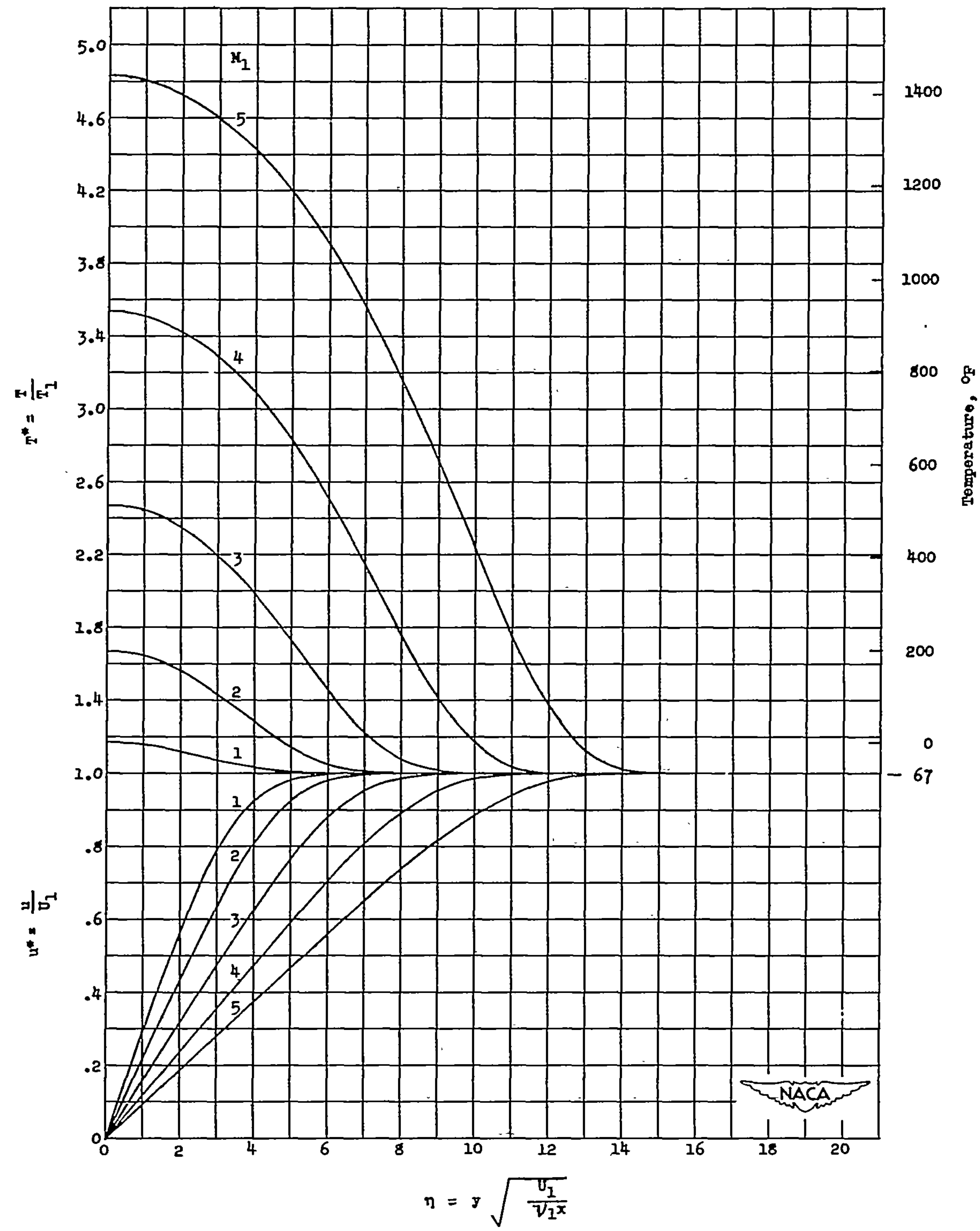

Figure 3.- Velocity and temperature profiles for the condition of zero heat transfer at the surface.

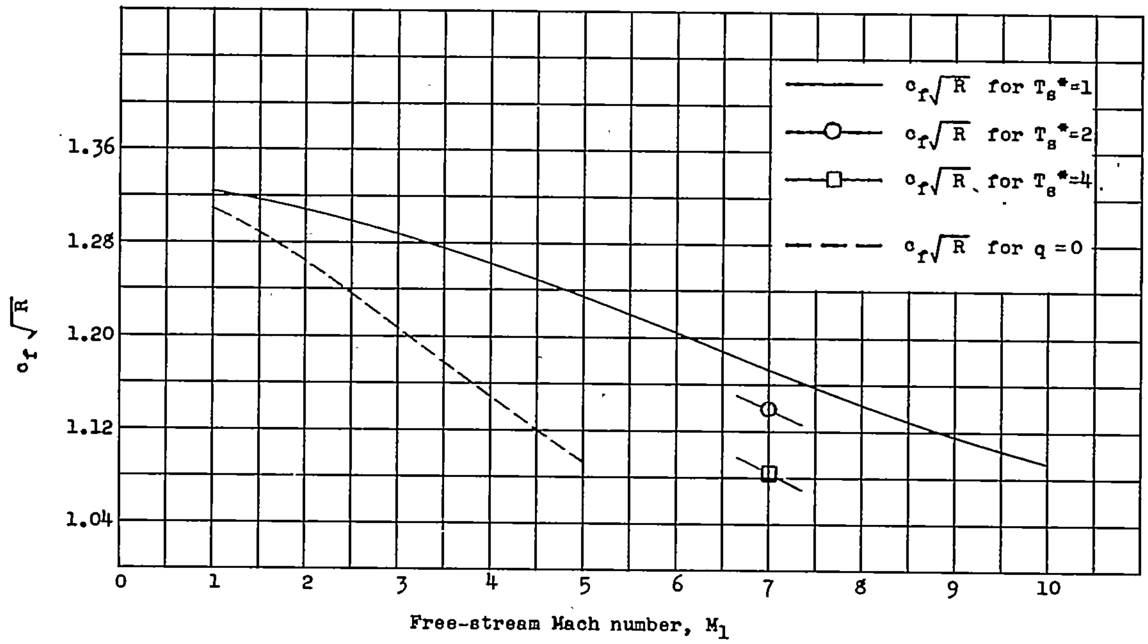

Figure 4.- Variation of skin-friction coefficient with stream Mach number.

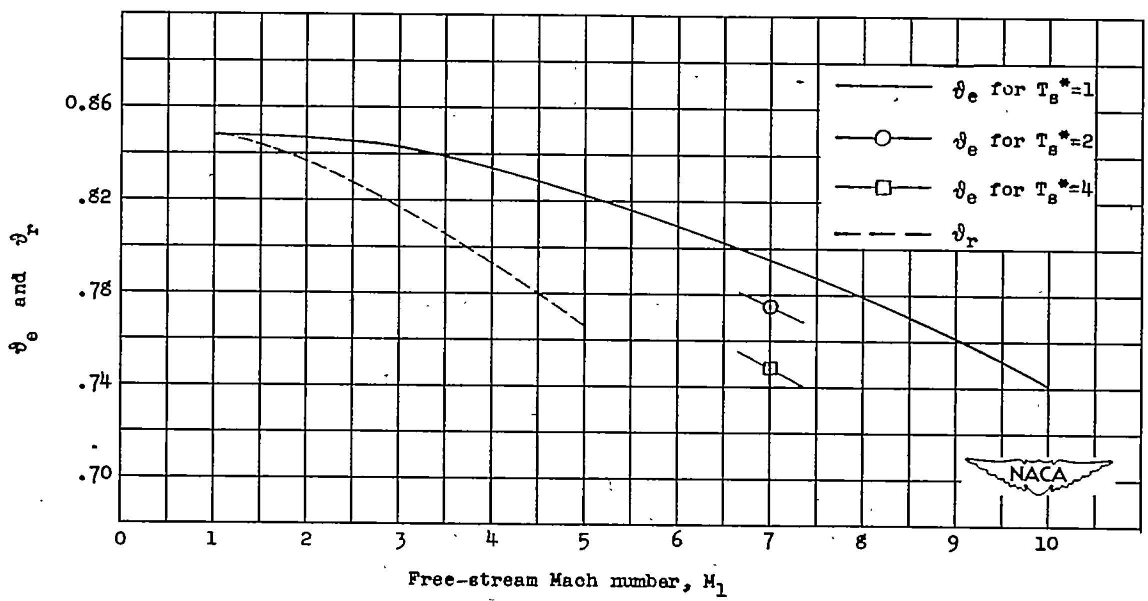

Figure 5.- Variation of effective temperature function and recovery factor with stream Mach number.

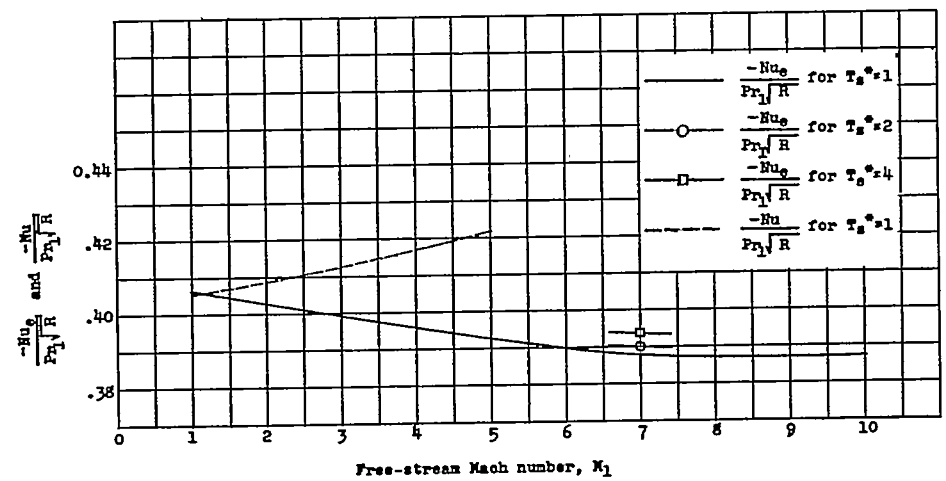

Figure 6.- Variation of Nusselt numbers, Nu_e and Nu, with stream Mach number.

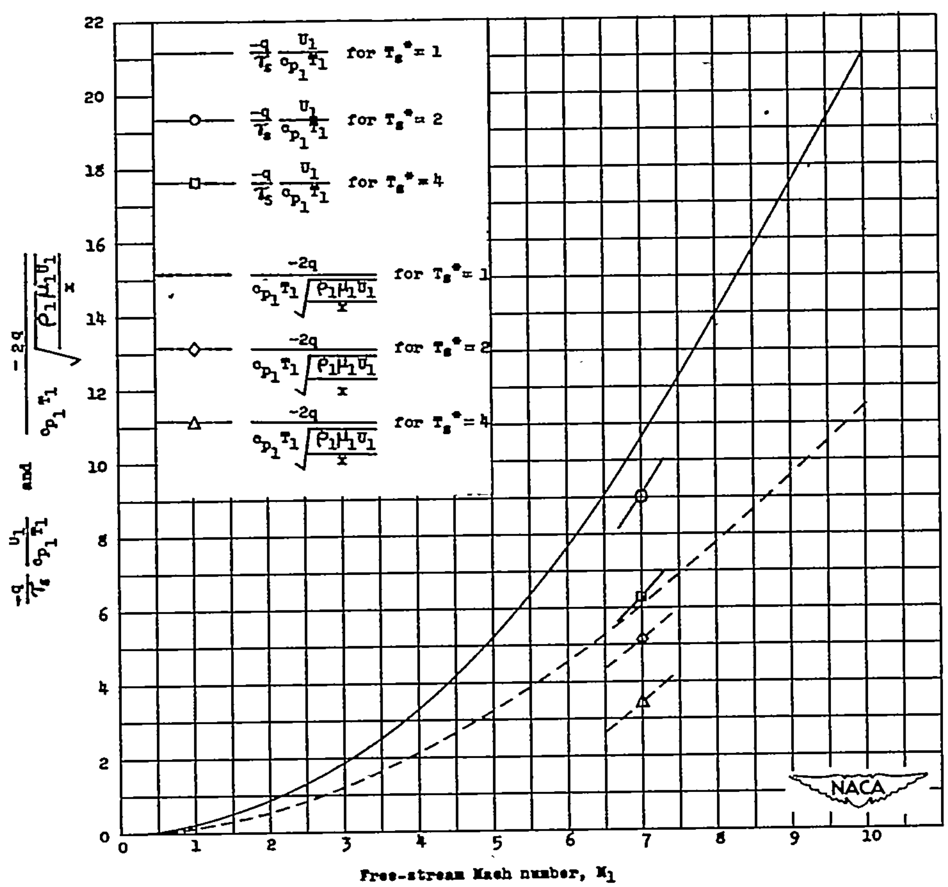

Figure 7.- Variation of the local heat-transfer rate and the ratio of heat transfer to shear stress with stream Mach number.